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Black-Scholes Value and Employee Stock Options
(Part I)

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Employee Stock Options (ESOs) have become an increasingly important part of executive compensation. Unfortunately, there are difficulties in determining precisely what they are worth. Recently there has been a push for accounting for ESOs as expenses in corporate financial statements, which has increased the focus on the question of what they are worth. However, it should be kept in mind that whatever solution is adopted for accounting purposes may not be the best way to look at ESOs for other purposes, such as deciding when to exercise one's ESOs.

By far the best known model for valuing stock options is the Black-Scholes model. It was developed by Fisher Black, Myron Scholes, and Robert Merton, and published in a paper written by Black and Scholes jointly and another written by Merton in 1973. Scholes and Merton received the Nobel Prize in economics in 1997 for this and related work; Black would undoubtedly have also been so honored if he had not died in 1995.

A *call* stock option is an option to buy a specified stock at a specified price, while a *put* stock option is an option to sell a specified stock at a specified price. The Black-Scholes formula is for the value of a call option. The value of a put option can be inferred from the value of the corresponding call option based on a concept called put-call parity. ESOs are always call options, however.

Assumptions

The Black-Scholes model makes certain assumptions, the most important of which for our purposes are that:

- The option is European, i.e., exercisable only at expiration.
- The stock does not pay dividends.
- The stock price follows a lognormal random walk process, also known as a geometric Brownian motion process, with drift.

Each of these assumptions is discussed in more detail below.

ESOs are American options, i.e., exercisable at any time, once they have vested. The value of an American option is always at least as large as that of a European option that is identical in all other respects. However, the difference in value is generally rather small. There will be no difference at all for a non-ESO option if the stock does not pay dividends, as under those circumstances it is always optimal to wait until expiration to exercise. The reason that this is optimal will be explained later, after the concept of *time value* has been introduced.

It can be optimal to exercise a non-ESO American option early if the stock pays dividends, but generally only if the dividend yield is high and the stock is about to pay its last dividend before expiration. The procedures used to calculate the value of an American option are considerable more complicated than calculating a Black-Scholes value (BSV), the user would have to input a dividend payment schedule, and the difference between the resulting value and the corresponding BSV would generally be rather small, so StockOpter *Insight* uses BSV rather than procedures designed to value American options.

The value of an option on a stock that pays dividends is less than the value would be if the stock did not pay dividends, all else being equal, provided that there is at least one dividend payment date before expiration. That is due to the stock price being expected to decrease by approximately the dividend payment when that payment is made. StockOpter *Insight* adjusts BSV for dividends, but it does so in an approximate way that requires as an input only the annual dividend payment amount, and not the dividend payment schedule. The approximation will work best when the option is relatively far from expiration.

The lognormal random walk model for the behavior of the price of a stock is an industry-standard model that has been found to work well in practice. It is based in part on the assumption that the stock market is efficient, i.e., that the stock price at a given moment reflects all the information available at that moment. Stock prices change for reasons, but changes that are about to happen will be due to new information, which by definition cannot be predicted ahead of time. It is of interest that the very activity of trading seems to be a major contributor to stock price volatility; evidently one of the pieces of information relevant to a stock price is what other market participants are doing. The "*with drift*" part of the description of the model refers to the fact that over time stock prices tend to increase. For a more detailed discussion of the lognormal random walk model please visit http://www.networthstrategies.com/.

For ESOs, the option owner might know significant information about his or her company not available to other investors, and thus have a somewhat better sense of where the stock price may be going than provided by the lognormal random walk model. ESO owners should try to avoid being overly confident of this, however.

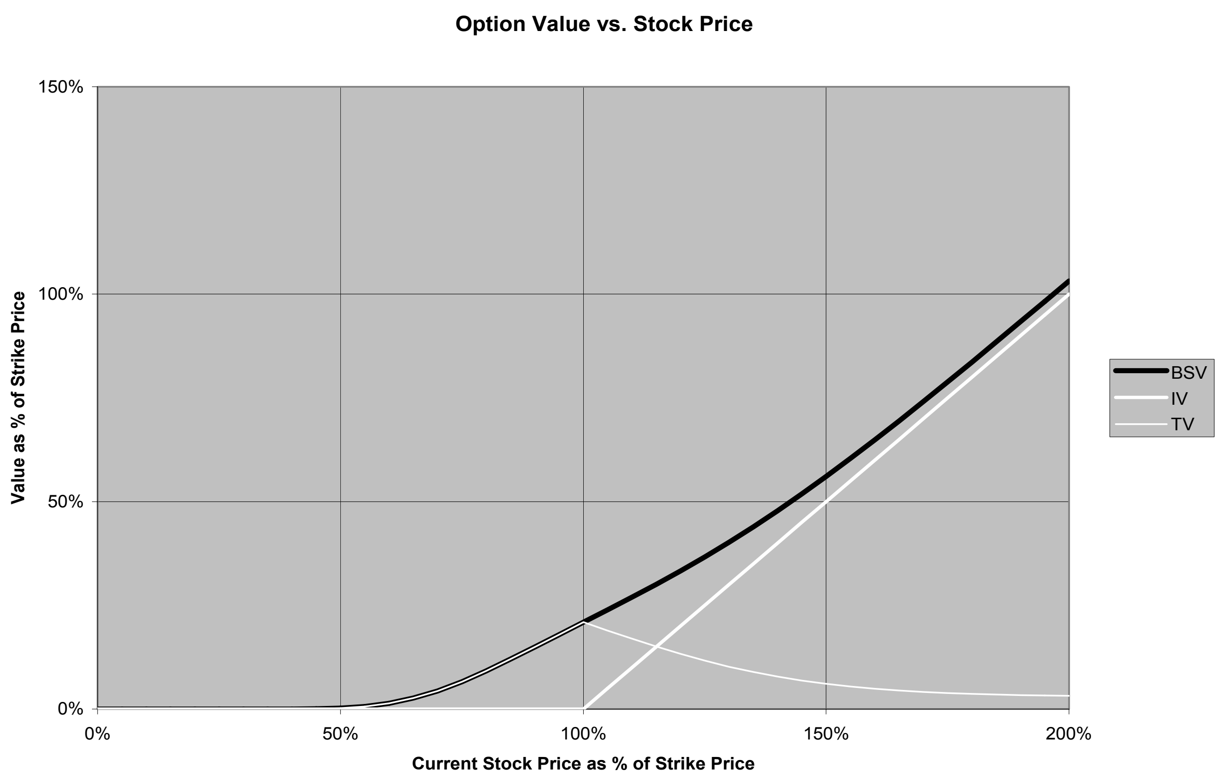
Intrinsic Value and Time Value

The BSV of an option can be thought of as consisting of two additive parts, namely its *intrinsic value* (IV) and its *time value* (TV). The IV is easily understood; it is the amount by which the option is in the money. It is the larger of the current stock price minus the strike price and zero. It is also known as the *in the money value*.

The TV is a little more complicated. Consider an option that is a little bit out of the money and is a ways away from maturity. Would you be willing to pay something for such an option, even though its IV is zero? Of course you would, since there is a significant chance that the option will be in the money at maturity. In this case what you would be willing to pay for is entirely TV.

Now, consider an option that is a little bit in the money and is a ways away from maturity. You now have some IV at risk, but not much. You could lose that, but you could also gain a lot if the stock price rises considerably. Thus, you would be willing to pay substantially more than the IV for the option, with the difference being the TV.

These ideas are illustrated in the following graph:



So long as the current price is less than the strike price (to the left of 100% in the above graph), the IV is zero. The higher the current price is, the higher the BSV, which in this range consists entirely of TV, will be. The TV is higher when the current stock price is higher in this range because the higher the price, both the more likely it is that the option will be in the money at expiration and the further in the money it is likely to be if it is in the money at expiration.

Once the current price becomes above the strike price, it continues to be true that the higher the current price is, the higher the BSV will be. However, in this range the increase in BSV is entirely due to increase in IV; the TV actually decreases as the current price rises. The TV decreasing is due to having more and more IV that can be lost if the price falls offsetting the potential gain in IV if the price rises. When the option is very far in the money, it is virtually certain that the option will expire in the money and will be exercised, and the economics become essentially the same as having irrevocably committed to buying the stock for the strike price at the expiration date. Here the only time value remaining is due to deferring paying the strike price. If one does not discount the strike price, the time value becomes essentially zero when the option is far in the money, as shown in the following graph:

If an option is a non-ESO American option on a stock that does not pay dividends, it will never be optimal to exercise early. If one does exercise early, one will receive only the IV, and not the TV. One may have any number of reasons for wanting to no longer own the option, but the optimal way to achieve that is to sell the option to someone else, and thus to receive the full BSV, not just the IV. This includes being able to sell an option that is out of the money, and for which the BSV thus consists entirely of TV.

Inputs to BSV and Their Effects

A BSV is calculated using these inputs:

- The option strike price (the price at which you can buy the stock if you exercise the option)
- The current stock price.
- The annualized continuously-compounded risk-free interest rate.
- The time until expiration in years.
- The annualized volatility of the stock price. This is a measure of the variability of the price of the stock. For a more detailed discussion of volatility please visit http://www.networthstrategies.com/.

The strike price and the current price are the sole determinants of the IV of an option. The relative values of these two inputs also affect the TV of an option, as described in the preceding section.

The risk-free interest rate affects the TV of an option in two ways:

- It is used for the expected rate of increase in the stock price.
- It is used to discount the probability-weighted average value of the option at expiration from the expiration date back to today.

One would think that for both of these purposes a risk premium should be added to the risk-free rate. That is correct in principle, but in practice this is not necessary because the error due leaving it out for one use cancels the error due to leaving it out for the other use. This is fortunate, because as a result one can ignore differences in risk appetite between different investors. The higher the risk-free interest rate is, the higher the TV will be, due to both of the ways in which it affects TV.

The annualized risk-free interest rate is multiplied by the time until expiration in years in all three places that it appears in the BSV equation. Thus, the longer the time until expiration, the higher the TV will be, for the reasons given above for the effects of the risk-free interest rate. However, the time until expiration also affects the TV for a third reason that is much more powerful than the other two. Time value comes predominately from the potential for change in the stock price from now to the expiration of the option. The longer the time until expiration, the greater the potential price change, and thus the larger the time value will be.

The effect of volatility on TV is closely related to the main effect of time until expiration, but the effect is even larger. The higher the volatility of the stock price, the greater the potential price change, and thus the larger the time value will be.

BSV and ESOs

There are a number of reasons why the true value of an ESO should be different from its BSV. These include:

- ESOs are American options, while BSV is for European options.
- BSV is for stocks that do not pay dividends.
- ESOs are subject to vesting.
- The time frame in which ESOs can be exercised generally becomes very short if the option owner leaves the company involuntarily.
- The option owner receives nothing for unexercised options if the option owner leaves the company voluntarily, so the option owner should exercise any vested options that are in the money before leaving.
- ESOs are not marketable.

The first two points above should have relatively small effects. The fact that ESOs are American options, while BSV is for European options, should make the value of an ESO higher than its BSV when this effect is considered in isolation. This effect generally will be small, however, and it will be zero if the stock does not pay dividends. As for dividends, StockOpter *Insight* uses a modified form of BSV that incorporates their effects, though in an approximate fashion.

The fact that ESOs are subject to vesting should have a dramatic impact on the values of options that are not yet vested and no impact on the values of options that are already vested. If one assumes that the option owner will stay with the company, the fact that the option owner cannot exercise the option right away would actually make the option intermediate between American and European, and thus improve the accuracy of using BSV. However, the option owner generally will receive nothing for the option if the option owner leaves the company. The value of an unvested option should be approximately its value if it were certain to vest times the probability that it will vest, but both of these inputs would be hard to calculate.

The option owner having to exercise his or her vested options more or less immediately or lose them if the option owner leaves the company reduces the value of a vested ESO. The option owner can receive the IV by exercising immediately, so this places a floor on the value of a vested ESO. BSV would be a good approximation if the option owner were certain to remain an employee until the expiration date and would not want to exercise early for some other reason, so BSV is an approximate ceiling on the value of a vested ESO. The actual value would be hard to calculate.

The fact that ESOs are not marketable means that any calculation of the value of an ESO cannot be empirically validated or invalidated by comparison to a value in the marketplace. Another result of ESOs not being marketable is that it may make sense to exercise early, as selling the option to someone else and thus receiving the TV, in addition to the IV that the option owner would receive in either case, is not an option. Reasons for exercising early include needing the money right away for an expense or wishing to diversify.

Given all of these limitations, why calculate BSV, and divide it into IV and TV? It is worth doing so because IV and TV can be useful inputs to exercise decisions. BSV will be, and IV can be, an overestimate of the value of an unvested ESO, but since it is unvested this is idle speculation anyway. What happens to these values is of interest to the option owner, but the option owner has no exercise decisions to make. Once an ESO has vested, however, the IV can be achieved by exercising, so it becomes important in the day-to-day decisions of whether to exercise. If the option owner exercises the ESO before its expiration date, he or she will in some sense lose the TV, so this also becomes important in the exercise decisions. The smaller the TV of an in-the-money option, the less the option owner is giving up by exercising early.

The value of the TV should be discounted in this deliberation if the option owner considers it likely that he or she will leave the company significantly before the ESO expiration date. One can experiment with this by substituting earlier dates for the actual expiration date and seeing the associated reduction in TV.